「電磁波工学の基礎」章末問題解答

＊解答が自明なものは省略します.

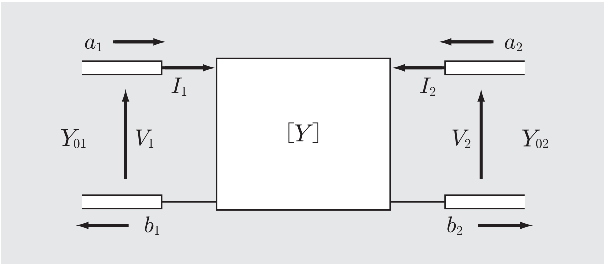
■ 1 章の問題

□ **1〜4**　省略

■ 2 章の問題

□ **1**　省略

□ **2**



$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \tag{1}$$

式 (2.86) より

$$\begin{aligned} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} &= \begin{pmatrix} \frac{1}{\sqrt{Y_{01}}}(a_1 + b_1) \\ \frac{1}{\sqrt{Y_{02}}}(a_2 + b_2) \end{pmatrix}, \\ \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} &= \begin{pmatrix} \sqrt{Y_{01}}(a_1 - b_1) \\ \sqrt{Y_{02}}(a_2 - b_2) \end{pmatrix} \end{aligned} \tag{2}$$

式 (2) を式 (1) へ代入して

$$\begin{pmatrix} \sqrt{Y_{01}}(a_1 - b_1) \\ \sqrt{Y_{02}}(a_2 - b_2) \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{Y_{01}}}(a_1 + b_1) \\ \frac{1}{\sqrt{Y_{02}}}(a_2 + b_2) \end{pmatrix}, \tag{3}$$

$$\begin{pmatrix} \sqrt{Y_{01}} & 0 \\ 0 & \sqrt{Y_{02}} \end{pmatrix} \begin{pmatrix} a_1 - b_1 \\ a_2 - b_2 \end{pmatrix} = \begin{pmatrix} Y \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{Y_{01}}} & 0 \\ 0 & \frac{1}{\sqrt{Y_{02}}} \end{pmatrix} \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}, \tag{4}$$

$$\begin{aligned} \begin{pmatrix} \sqrt{Y_{01}} & 0 \\ 0 & \sqrt{Y_{02}} \end{pmatrix}^{-1} &= \frac{1}{\sqrt{Y_{01}Y_{02}}} \begin{pmatrix} \sqrt{Y_{02}} & 0 \\ 0 & \sqrt{Y_{01}} \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{\sqrt{Y_{01}}} & 0 \\ 0 & \frac{1}{\sqrt{Y_{02}}} \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{01}} & 0 \\ 0 & \sqrt{Z_{02}} \end{pmatrix}, \end{aligned} \tag{5}$$

だから，式 (5) を式 (4) の左からかけて

$$
\begin{aligned}
&[a]-[b]=\left[\sqrt{Z_0}\right][Y]\left[\sqrt{Z_0}\right]\{[a]+[b]\},\\
&[\hat{Y}]\triangleq\left[\sqrt{Z_0}\right][Y]\left[\sqrt{Z_0}\right]
\end{aligned}
$$

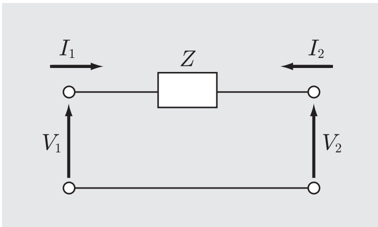
とすれば,

$$
\begin{aligned}
&[a]-[b]=[\hat{Y}]\{[a]+[b]\},\\
&\{[1]-[\hat{Y}]\}[a]=\{[1]+[\hat{Y}]\}[b]
\end{aligned}
$$

ゆえに

$$
[b]=\underbrace{\{[1]+[\hat{Y}]\}^{-1}\{[1]-[\hat{Y}]\}}_{[S]}[a]
$$

□ 3



$$
\begin{cases}I_1=-I_2\\ V_1=I_1Z+V_2\end{cases}\quad\Longrightarrow\quad\begin{cases}I_1=\frac{I}{Z}(V_1-V_2)\\ I_2=-\frac{I}{Z}(V_1-V_2)\end{cases}
$$

ゆえに

$$
\begin{aligned}
&\begin{pmatrix}I_1\\ I_2\end{pmatrix}=\frac{1}{Z}\begin{pmatrix}1&-1\\ -1&1\end{pmatrix}\begin{pmatrix}V_1\\ V_2\end{pmatrix},\\
&[\hat{Y}]=\left[\sqrt{Z_0}\right][Y]\left[\sqrt{Z_0}\right]\\
&\quad=\begin{pmatrix}\sqrt{Z_0}&0\\ 0&\sqrt{Z_0}\end{pmatrix}\frac{1}{Z}\begin{pmatrix}1&-1\\ -1&1\end{pmatrix}\begin{pmatrix}\sqrt{Z_0}&0\\ 0&\sqrt{Z_0}\end{pmatrix}\\
&\quad=\frac{Z_0}{Z}\begin{pmatrix}1&0\\ 0&1\end{pmatrix}\begin{pmatrix}1&-1\\ -1&1\end{pmatrix}\begin{pmatrix}1&0\\ 0&1\end{pmatrix}=\frac{1}{\hat{Z}}\begin{pmatrix}1&-1\\ -1&1\end{pmatrix}
\end{aligned}
$$

よって,

$$[S] = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{\hat{Z}} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \right\}^{-1} \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{1}{\hat{Z}} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \right\}$$

$$= \begin{pmatrix} 1+\frac{1}{\hat{Z}} & -\frac{1}{\hat{Z}} \\ -\frac{1}{\hat{Z}} & 1+\frac{1}{\hat{Z}} \end{pmatrix}^{-1} \begin{pmatrix} 1-\frac{1}{\hat{Z}} & \frac{1}{\hat{Z}} \\ \frac{1}{\hat{Z}} & 1-\frac{1}{\hat{Z}} \end{pmatrix}$$

$$= \frac{1}{\left(1+\frac{1}{\hat{Z}}\right)^2 - \left(\frac{1}{\hat{Z}}\right)^2} \begin{pmatrix} 1+\frac{1}{\hat{Z}} & \frac{1}{\hat{Z}} \\ \frac{1}{\hat{Z}} & 1+\frac{1}{\hat{Z}} \end{pmatrix}^{-1} \begin{pmatrix} 1-\frac{1}{\hat{Z}} & \frac{1}{\hat{Z}} \\ \frac{1}{\hat{Z}} & 1-\frac{1}{\hat{Z}} \end{pmatrix}$$

$$= \frac{1}{1+\frac{1}{\hat{Z}}} \begin{pmatrix} 1 & \frac{2}{\hat{Z}} \\ \frac{2}{\hat{Z}} & 1 \end{pmatrix}$$

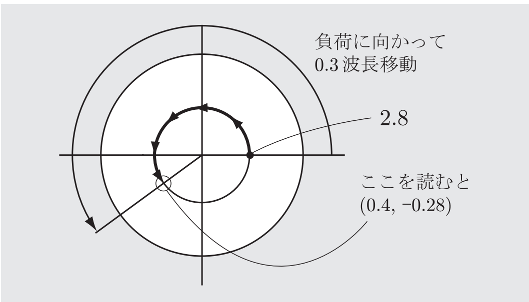
$$= \frac{1}{\hat{Z}+2} \begin{pmatrix} \hat{Z} & 2 \\ 2 & \hat{Z} \end{pmatrix}$$

□ **4**　(1)　線路上の波長は

$$\lambda_g = \frac{3 \times 10^8}{2 \times 10^9} \times \frac{1}{\sqrt{6.25}} = 60\,[\mathrm{mm}]$$

従って 18 mm は

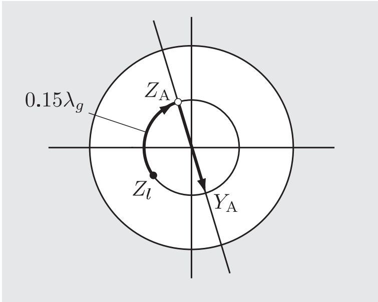
$$\frac{18}{60} = 0.3\,[\text{波長}]$$



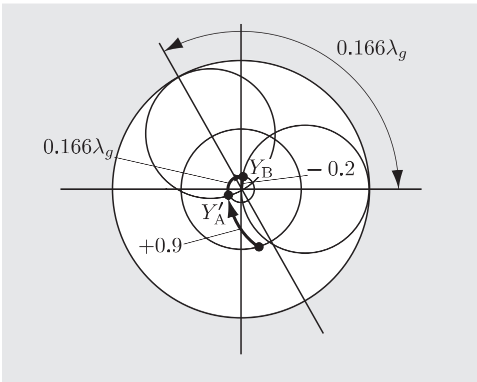
ゆえに,アンテナインピーダンスは

$$75 \times (0.4 - j0.28) = 30 - j21\,[\Omega]$$

(2)　9 mm → $0.15\lambda_g$,10 mm → $0.166\lambda_g$.スミスチャート上で $Z_l \rightarrow\ Z_{\mathrm{A}} \rightarrow\ Y_{\mathrm{A}}$ と進む.



Y_A に $+0.9$ のサセプタンスをつなぎ，Y_A' へ移動．そこから $0.166\,\lambda_g$（10 mm）さらにさかのぼり，$G=1$ の円上に到達できる（Y_B）．そこで，-0.2 のサセプタンスをつなぐと，原点へ到達する．

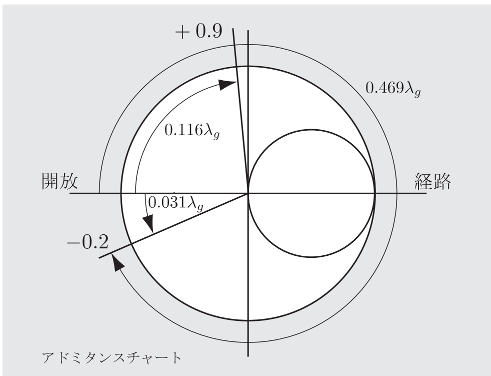


ゆえに

A 点に　$0.469\times 60\,[\mathrm{mm}] = 28\,[\mathrm{mm}]$　（-0.2 の正規化サセプタンスに相当）

B 点に　$0.116\times 60\,[\mathrm{mm}] = 7\,[\mathrm{mm}]$　（$+0.9$ の正規化サセプタンスに相当）

の終端開放線路をつなげばよい．



■ 3 章の問題

□ **1**　式 (3.44) の導出.

$$\boldsymbol{S} \triangleq \boldsymbol{E} \times \boldsymbol{H}^* \qquad \text{(複素ポインティングベクトル)}$$

$$\nabla \cdot \boldsymbol{S} = \nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}^*) = \boldsymbol{H}^* \cdot \nabla \times \boldsymbol{E} - \boldsymbol{E} \cdot \nabla \times \boldsymbol{H}^*$$

$$\overset{(3.14),(3.15)}{=} \boldsymbol{H}^* \cdot (-j\omega\mu\boldsymbol{H}) - \boldsymbol{E} \cdot (\sigma - j\omega\varepsilon)\boldsymbol{E}^*$$

$$= -j\omega\mu|\boldsymbol{H}|^2 - \sigma|\boldsymbol{E}|^2 + j\omega\varepsilon|\boldsymbol{E}|^2$$

ゆえに

$$\int \nabla \cdot \boldsymbol{S} \mathrm{d}V = -\int_V \sigma|\boldsymbol{E}|^2 \mathrm{d}V + j\omega \int_V (\varepsilon|\boldsymbol{E}|^2 - \mu|\boldsymbol{H}|^2)\mathrm{d}V$$

一方,

$$\int_V \nabla \cdot \boldsymbol{S} \mathrm{d}V \overset{\text{ガウスの定理}}{=} \int_S \boldsymbol{S} \mathrm{d}\boldsymbol{S} = \int_S (\boldsymbol{E} \times \boldsymbol{H}^*) \mathrm{d}\boldsymbol{S}$$

よって

$$\int_S (\boldsymbol{E} \times \boldsymbol{H}^*) \mathrm{d}\boldsymbol{S} + \int_V \sigma|\boldsymbol{E}|^2 \mathrm{d}V = j\omega \int_V (\varepsilon|\boldsymbol{E}|^2 - \mu|\boldsymbol{H}|^2)\mathrm{d}V$$

□ **2**　省略

□ **3**

● 遮断周波数

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi}ck_c = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2} \quad (\mathrm{TM}_{11} \ \rightarrow \ m = n = 1)$$

● 位相定数

$$\beta = \sqrt{\omega^2\varepsilon\mu - k_c^2} = \sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2} - \frac{\pi^2}{b^2}} \quad \left(\text{または } = \pi\sqrt{\left(\tfrac{2f}{c}\right)^2 - \tfrac{1}{a^2} - \tfrac{1}{b^2}}\ \right)$$

● 管内波長

$$\lambda_g = \frac{2\pi}{\beta} = \frac{2}{\sqrt{\left(\frac{2f}{c}\right)^2 - \frac{1}{a^2} - \frac{1}{b^2}}} \quad \left(\text{または } = \frac{2\pi}{\sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2} - \frac{\pi^2}{b^2}}}\ \right)$$

● 位相速度

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2} - \frac{\pi^2}{b^2}}} \quad \left(\text{または } = \frac{1}{\sqrt{\frac{1}{c^2} - \frac{1}{\omega^2}\left(\frac{\pi^2}{a^2} + \frac{\pi^2}{b^2}\right)}}\ \right)$$

● 群速度

$$v_g = \left(\frac{\mathrm{d}\beta}{\mathrm{d}\omega}\right)^{-1} = \left(\frac{1}{2}(\omega^2\varepsilon\mu - k_c^2)^{-\frac{1}{2}} 2\omega\varepsilon\mu\right)^{-1}$$

$$= \frac{\sqrt{\omega^2\varepsilon\mu - k_c^2}}{\omega\varepsilon\mu} = \frac{\sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2} - \frac{\pi^2}{b^2}}}{\omega\varepsilon\mu} = \frac{c^2}{\omega}\sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2} - \frac{\pi^2}{b^2}}$$

$$\left(\text{または } = \frac{c^2}{2f}\sqrt{\frac{4f^2}{c^2} - \frac{1}{a^2} - \frac{1}{b^2}}\ \right)$$

● 伝送電力

$$
\begin{aligned}
\boldsymbol{k} \cdot S_z &= \boldsymbol{E}_t \times \boldsymbol{H}_t^* = (\boldsymbol{i}E_x + \boldsymbol{j}E_y) \times (\boldsymbol{i}H_x^* + \boldsymbol{j}H_y^*) = \boldsymbol{k}(E_x H_y^* - E_y H_x^*) \\
&= \boldsymbol{k}\Bigg(-\frac{\gamma k_x}{k_c^2} E_{11} \cos k_x x \sin k_y y \cdot \frac{j\omega\varepsilon k_x}{k_c^2} E_{11}^* \cos k_x x \sin k_y y \\
&\qquad - (-)\frac{\gamma k_y}{k_c^2} E_{11} \sin k_x x \cos k_y y \cdot (-)\frac{j\omega\varepsilon k_y}{k_c^2} E_{11}^* \sin k_x x \cos k_y y \Bigg) \\
&= \boldsymbol{k}\left(\frac{\beta\omega\varepsilon k_x^2}{k_c^4} |E_{11}|^2 \cos^2 k_x x \sin^2 k_y y + \frac{\beta\omega\varepsilon k_y^2}{k_c^4} |E_{11}|^2 \sin^2 k_x x \cos^2 k_y y \right) \\
&= \boldsymbol{k}\left(\frac{\beta\omega\varepsilon |E_{11}|^2}{k_c^4} (k_x^2 \cos^2 k_x x \sin^2 k_y y + k_y^2 \sin^2 k_x x \cos^2 k_y y) \right) \\
&= \boldsymbol{k}\frac{\beta\omega\varepsilon |E_{11}|^2}{k_c^4} \left(\left(\frac{\pi}{a}\right)^2 \cos^2 \frac{\pi x}{a} \sin^2 \frac{\pi y}{b} + \left(\frac{\pi}{b}\right)^2 \sin^2 \frac{\pi x}{a} \cos^2 \frac{\pi y}{b} \right)
\end{aligned}
$$

$$
\begin{aligned}
\therefore \quad P &= \int_0^a \int_0^b \mathrm{Re}\ S_z \mathrm{d}x\mathrm{d}y \\
&= \frac{\beta\omega\varepsilon |E_{11}|^2}{k_c^4} \Bigg[\left(\frac{\pi}{a}\right)^2 \int_0^a \int_0^b \cos^2 \frac{\pi x}{a} \sin^2 \frac{\pi y}{b} \mathrm{d}x\mathrm{d}y \\
&\qquad + \left(\frac{\pi}{b}\right)^2 \int_0^a \int_0^b \sin^2 \frac{\pi x}{a} \cos^2 \frac{\pi y}{b} \mathrm{d}x\mathrm{d}y \Bigg] \\
&= \frac{\beta\omega\varepsilon |E_{11}|^2}{\left(\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 \right)^2} \int_0^a \int_0^b \Bigg\{ \left(\frac{\pi}{a}\right)^2 \cos^2 \frac{\pi x}{a} \sin^2 \frac{\pi y}{b} \\
&\qquad + \left(\frac{\pi}{b}\right)^2 \sin^2 \frac{\pi x}{a} \cos^2 \frac{\pi y}{b} \Bigg\} \mathrm{d}x\mathrm{d}y
\end{aligned}
$$

● 特性インピーダンス

$$
\mathrm{TM} \ : \ Z_E = \frac{\beta}{\omega\varepsilon} = \frac{1}{\omega\epsilon}\sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2} - \frac{\pi^2}{b^2}}
$$

□ **4** (1) C バンドは 4〜8 GHz．矩形導波管の辺を a, b として，遮断周波数は式 (3.82) より $f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$.

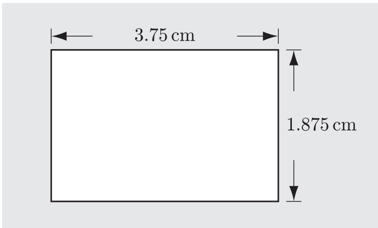
m \ n	0	1	2
0	-	$\frac{c}{2b}$	$\frac{2c}{2b}$
1	$\frac{c}{2a}$	$\frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$	$\frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{2}{b}\right)^2}$
2	$\frac{2c}{2a}$	$\frac{c}{2}\sqrt{\left(\frac{2}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$	$\frac{c}{2}\sqrt{\left(\frac{2}{a}\right)^2 + \left(\frac{2}{b}\right)^2}$

ゆえに，4〜8 GHz で TE_{10} の単一モード導波管とするには

$$\frac{c}{2a} = 4 \times 10^9, \qquad \frac{c}{2b} = 8 \times 10^9$$

とすればよい．

$$\therefore\ a = \frac{3 \times 10^8}{2 \times 4 \times 10^9} = 3.75\,[\mathrm{cm}], \quad \therefore\ b = \frac{3 \times 10^8}{2 \times 8 \times 10^9} = 1.875\,[\mathrm{cm}]$$



(2)　$\frac{4+8}{2} = 6\,\mathrm{GHz}$ での管内波長 λ_g，特性インピーダンス Z_0，群速度 v_g を求める．

$$\lambda = \frac{3 \times 10^8}{6 \times 10^9} = 5\,[\mathrm{cm}], \qquad \lambda_c = \frac{3 \times 10^8}{4 \times 10^9} = 7.5\,[\mathrm{cm}]$$

式 (3.96) より

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \frac{\lambda}{\lambda_c}}} = \frac{5}{\sqrt{1 - \left(\frac{5}{7.5}\right)^2}} = 6.71\,[\mathrm{cm}]$$

式 (3.104) より

$$Z_0 = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{j\beta} = \frac{\omega\mu}{\beta} = \frac{2\pi f\mu}{\frac{2\pi}{\lambda_g}} = \lambda_g \cdot f \cdot \mu$$

$$= 6.71 \times 10^{-2} \times 6 \times 10^9 \times \underbrace{4\pi \cdot 10^{-7}}_{\text{式 (3.33)}} = 505.8\,[\Omega]$$

$$v_g = \frac{c^2}{v_p} = \frac{c^2}{f \cdot \lambda_g} = \frac{(3 \times 10^8)^2}{(6 \times 10^9) \cdot (6.71 \times 10^{-2})} = 2.24 \times 10^8\,[\mathrm{m/s}]$$

(3)　X バンドは 8〜12 GHz．$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$ に $a = 3.75\,[\mathrm{cm}], b = 1.875\,[\mathrm{cm}]$ を代入して，下表を得る．

m ＼ n	0	1	2	3
0	-	8**	16	24
1	4*	8.94**	16.49	24.33
2	8**	11.3**	17.9	25.3
3	12	14.4	20	26.8

従って，TE_{10}（$*$）に加え，$\mathrm{TE}_{01}, \mathrm{TE}_{11}, \mathrm{TM}_{11}, \mathrm{TE}_{20}, \mathrm{TE}_{21}, \mathrm{TM}_{21}$（$**$）が励振され得る．従って，X バンドでは7つのモードが存在し得る．

□ **5**　(1)

$$Z_0 = 138 \log_{10} \frac{b}{a} = 138 \log_{10} \frac{\frac{d}{2}}{\frac{2}{2}} = 138 \log_{10} \frac{d}{2} \implies 100\,[\Omega]$$

$$\therefore\ \frac{100}{138} = \log_{10} \frac{d}{2}, \quad \therefore\ 10^{\frac{100}{138}} = \frac{d}{2}, \quad \therefore\ d = 2 \times 10^{\frac{100}{138}} = 10.6\,[\mathrm{mm}]$$

(2)

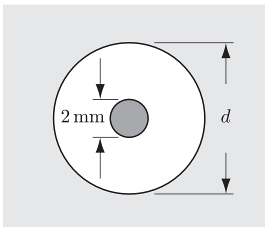
$$f_c \cong \frac{c}{\pi} \cdot \frac{1}{a+b} = \frac{c}{\pi} \cdot \frac{1}{\left(\frac{2}{2} + \frac{10.6}{2}\right) \times 10^{-3}} = 15.2\,[\mathrm{GHz}]$$

(3)

$$E_r\Big|_a = \frac{V_0}{a} = 2 \times 10^5\,[\mathrm{V/m}], \qquad \therefore\ V_0 = 2 \times 10^5 \times \underbrace{\frac{2}{2} \times 10^{-3}}_{a} = 2 \times 10^2$$

$$\therefore\ P = \frac{2\pi}{\zeta} |V_0|^2 \ln \frac{b}{a} = \frac{2\pi}{377} |200|^2 \ln \frac{\frac{10.6}{2}}{\frac{2}{2}} = 1.1 \times 10^3\,[\mathrm{W}] = 1.1\,[\mathrm{kW}]$$

（平均ではこの半分の 555 W．どちらも正解）

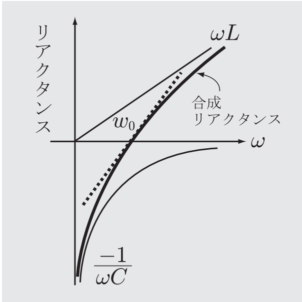


□ **6**　図 3.34，図 3.33 より $\frac{a}{b} = 3$ の Z_0 と $\sqrt{\varepsilon_{\mathrm{eff}}}$ を求めると，各々，$70\,\Omega$ と 2.5．よって特性インピーダンスは $\frac{70}{2.5} = 30\,[\Omega]$．

■ 4 章の問題

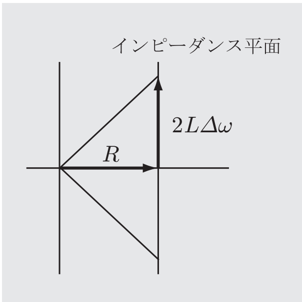
□ **1**

$$\text{共振：}\quad \omega_0 L - \frac{1}{\omega_0 C} = 0 \longrightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$



上図の破線で示した直線（接線）は

$$\frac{\mathrm{d}}{\mathrm{d}\omega}\left(\omega L-\frac{1}{\omega C}\right)\bigg|_{\omega=\omega_0}\cdot\underset{(\omega-\omega_0)}{\Delta\omega}=\left(L+\frac{1}{C}\omega^{-2}\right)\bigg|_{\omega=\omega_0}\cdot\Delta\omega=2L\Delta\omega$$



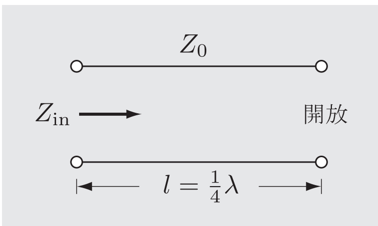
図より

$$2L\Delta\omega=R,\qquad\therefore\ \Delta\omega_0=\frac{R}{2L}$$

よって

$$Q\triangleq\frac{\omega_0}{2\Delta\omega_0}=\frac{1}{\sqrt{LC}}\cdot\frac{1}{2}\cdot\frac{2L}{R}=\frac{1}{R}\sqrt{\frac{L}{C}}$$

□ **2**



式 (2.73) より

$$Z_{\mathrm{in}}=-jZ_0\cot\beta l$$

一般に，$\coth Z = \frac{1}{\gamma}\cot\frac{Z}{j}$ であることから

$$Z_{\mathrm{in}} = Z_0 \coth \gamma l$$

これに，$l = \frac{\lambda_{g0}}{4} = \frac{1}{4}\cdot\frac{2\pi C}{\omega_0}$ を代入すると $\beta l = \frac{\pi}{2}\left(1+\frac{\Delta\omega}{\omega_0}\right)$ なので

$$\begin{aligned}
Z_{\mathrm{in}} = Z_0 \coth\gamma l &= Z_0 \coth\left(\alpha l + j\frac{\pi}{2}\left(1+\frac{\Delta\omega}{\omega_0}\right)\right) \\
&= Z_0 \coth\left(\alpha l + j\frac{\pi}{2}\cdot\frac{\Delta\omega}{\omega_0} + j\frac{\pi}{2}\right) \\
&= Z_0 \tanh\left(\alpha l + j\frac{\pi}{2}\cdot\frac{\Delta\omega}{\omega_0}\right) \\
&\cong Z_0 \,\mathrm{sech}^2(0)\cdot\left(\alpha l + j\frac{\pi}{2}\cdot\frac{\Delta\omega}{\omega_0}\right) \quad (\text{if} \propto 小 : \Delta\omega \ \to \ 0) \\
&= Z_0\left(\alpha l + j\frac{\pi}{2}\cdot\frac{\Delta\omega}{\omega_0}\right) \\
&= \frac{\pi\alpha}{2\beta_0}Z_0\left(1 + j\frac{\beta_0}{2\alpha}\left(\frac{2\Delta\omega}{\omega_0}\right)\right) \quad (l = \tfrac{\pi}{2\beta_0}\text{を代入})
\end{aligned}$$

これを式 (4.2) と比較すると（$\frac{\beta_0}{2\alpha} \triangleq Q$ として）

$$Z_{\mathrm{in}} = \frac{\pi Z_0}{4Q}\left(1 + jQ\frac{2\Delta\omega}{\omega_0}\right)$$

を得る．等価回路の対応は

$$\begin{cases}
R = \dfrac{\pi Z_0}{4Q} \\
L = \dfrac{QR}{\omega_0} = \dfrac{\pi Z_0}{4\omega_0} \\
C = \dfrac{1}{\omega_0^2 L} = \dfrac{4\omega_0}{\pi Z_0}\cdot\dfrac{1}{\omega_0^2} = \dfrac{4}{\omega_0 \pi Z_0}
\end{cases}$$

となる．

□ **3〜6**　省略

□ **7**　TE_{10n} について，式 (4.61) より，共振波長は $\frac{1}{\left(\frac{\lambda}{2}\right)^2} = \frac{1}{a^2} + \frac{1}{\left(\frac{c}{n}\right)^2}$ で決まる．共振周波数に直すと

$$f = \frac{光速}{\lambda} = \frac{光速}{2}\sqrt{\frac{1}{a^2} + \frac{1}{\left(\frac{c}{n}\right)^2}} \tag{4.51}$$

今，$f = 1.2 f_c$ なので

$$\frac{光速}{2}\sqrt{\frac{1}{a^2} + \frac{1}{\left(\frac{c}{n}\right)^2}} = \frac{光速}{2a}\cdot 1.2$$

$$\therefore\ \frac{(1.2)^2}{a^2} = \frac{1}{a^2} + \frac{1}{\left(\frac{c}{n}\right)^2}, \quad \therefore\ \left(\frac{c}{n}\right)^2 = \frac{a^2}{(1.2)^2 - 1}$$

ゆえに

$$\frac{c}{n} = \frac{a}{\sqrt{(1.2)^2 - 1}} = \frac{a}{0.663} = \frac{5.7}{0.663} = 8.59\,[\mathrm{mm}]$$

よって，$\underline{\mathrm{TE}_{101}\text{ について } c = 8.59\,[\mathrm{mm}]}$ とすればよい．

□ **8**　(1)　図 3.34 より $\frac{a}{b} = 2$ で $\varepsilon_r = 1$ の Z_0 を読むと 90 Ω．一方，図 3.33 で，$\frac{a}{b} = 2$ で $\varepsilon_r = 4$ の $\sqrt{\varepsilon_{\text{eff}}}$ を読むと $\underline{1.8}$．ゆえに

$$\sqrt{\varepsilon_{\text{eff}}} = \underline{3.24}$$

よって

$$\frac{90}{1.8} = \underline{50\,[\Omega]}$$

(2)　式 (4.28) より

$$\frac{\lambda_g}{2} - 2v_p Z_0 C = l$$

ゆえに

$$\begin{aligned}\lambda_g &= 2(l + 2v_p Z_0 C) = 2(7.5 \times 10^{-3} + 2 \times \frac{3 \times 10^8}{1.8} \times 50 \times 0.05 \times 10^{-12}) \\ &= 16.7\,[\mathrm{mm}]\end{aligned}$$

ゆえに

$$f = \frac{C}{\sqrt{\varepsilon_{\text{eff}}}\ \lambda_g} = \frac{3 \times 10^8}{1.8 \times 16.7 \times 10^{-3}} = \underline{10\,[\mathrm{GHz}]}$$

(3)(4)　下図参照．