工科のための 複　素　解　析　演習問題略解

第 1 章　1.3　(1) $\sqrt{2}$　(2) 2　(3) $\sqrt{10}$　(4) $\sqrt{10}$　(5) $2\sqrt{2}$　(6) $\dfrac{\sqrt{13}}{6}$　(7) $\sqrt{\dfrac{11}{6}}$　(8) 4

1.4　(1) $7(1-i)$　(2) $\dfrac{1}{6}\big(-11+13i\big)$　(3) $1+13i$　(4) $\big(2\sqrt{2}+\sqrt{3}\big)+i\big(2-\sqrt{6}\big)$　(5) $-(119+120i)$

(6) $\dfrac{1}{13}(-6+7i)$　(7) $-\dfrac{1}{4}\big\{\big(\sqrt{3}-\sqrt{2}\big)+i\big(\sqrt{2}+\sqrt{3}\big)\big\}$　(8) $\dfrac{1}{625}\big\{\big(70-216\sqrt{3}\,\big)-i\big(240+63\sqrt{3}\,\big)\big\}$

(9) $-\dfrac{1}{2}(3+i)$　(10) $\dfrac{1}{5}(-1+7i)$　(11) $\dfrac{1}{4}\big\{-\big(2\sqrt{3}+1\big)+\big(2-\sqrt{3}\big)\big\}$　(12) $\dfrac{3}{173}(16-69i)$

1.5　(1) $\pm(2-i)$　(2) $\pm(3+2i)$　(3) $\pm(4-i)$　(4) $\pm(4+3i)$　(5) $\pm\dfrac{1}{2}\big\{\big(\sqrt{3}-1\big)-i\big(\sqrt{3}+1\big)\big\}$

(6) $\pm\dfrac{\sqrt[4]{3}}{2}\big\{\big(\sqrt{3}+1\big)+i\big(\sqrt{3}-1\big)\big\}$　(7) $\pm\pm\left(\sqrt{\sqrt{2}+1}+i\sqrt{\sqrt{2}-1}\,\right)$

(8) $\pm\left(\sqrt{\sqrt{2}-1}+i\sqrt{\sqrt{2}+1}\,\right)$

1.6　(1) $\pm\sqrt{2}\sqrt{\sqrt{2}-1}-i\left(1\mp\sqrt{2}\sqrt{\sqrt{2}+1}\,\right)$　(2) $\pm\left\{\sqrt{\sqrt{3}+\sqrt{2}}+i\left(\sqrt[4]{2}\pm\sqrt{\sqrt{3}-\sqrt{2}}\,\right)\right\}$

(3) $\left(2\pm\sqrt{\sqrt{2}+1}\right)+i\left(1\pm\sqrt{\sqrt{2}-1}\,\right)$　(4) $\big(\sqrt{2}\pm\sqrt{3}\,\big)+i\big(\sqrt{3}\pm\sqrt{2}\,\big)$

1.8　(1) $\sqrt{2}\left(\cos\dfrac{\pi}{4}+i\sin\dfrac{\pi}{4}\right)$　(2) $2\sqrt{2}\left(\cos\dfrac{3\pi}{4}+i\sin\dfrac{3\pi}{4}\right)$　(3) $\dfrac{1}{\sqrt{2}}\left\{\cos\left(-\dfrac{\pi}{4}\right)+i\sin\left(-\dfrac{\pi}{4}\right)\right\}$

(4) $5\left\{\cos\left(-\dfrac{3\pi}{4}\right)+i\sin\left(-\dfrac{3\pi}{4}\right)\right\}$ (5) $2\left\{\cos\left(-\dfrac{\pi}{6}\right)+i\sin\left(-\dfrac{\pi}{6}\right)\right\}$

(6) $2\sqrt{3}\left\{\cos\left(-\dfrac{2\pi}{3}\right)+i\sin\left(-\dfrac{2\pi}{3}\right)\right\}$　(7) $2\sqrt{2}\left(\cos\dfrac{\pi}{6}+i\sin\dfrac{\pi}{6}\right)$　(8) $3\left\{\cos\left(-\dfrac{\pi}{3}\right)+i\sin\left(-\dfrac{\pi}{3}\right)\right\}$

(9) $\sqrt{2}\left(\cos\dfrac{5\pi}{6}+i\sin\dfrac{5\pi}{6}\right)$　(10) $\dfrac{1}{\sqrt{2}}\left(\cos\dfrac{7\pi}{12}+i\sin\dfrac{7\pi}{12}\right)$

(11) $(\sqrt{6}+\sqrt{2})\left\{\cos\left(-\dfrac{7\pi}{12}\right)+i\sin\left(-\dfrac{7\pi}{12}\right)\right\}$　(12) $(\sqrt{6}+\sqrt{2})\left\{\cos\left(-\dfrac{7\pi}{12}\right)+i\sin\left(-\dfrac{7\pi}{12}\right)\right\}$

(13) $\sqrt{4+2\sqrt{2}}\left\{\cos\left(-\dfrac{\pi}{8}\right)+i\sin\left(-\dfrac{\pi}{8}\right)\right\}$　(14) $2\sqrt{12+6\sqrt{2}}\left\{\cos\left(-\dfrac{\pi}{24}\right)+i\sin\left(-\dfrac{\pi}{24}\right)\right\}$

(15) $\dfrac{\sqrt{2+\sqrt{2}}}{\sqrt{3}+1}\left(\cos\dfrac{5\pi}{24}+i\sin\dfrac{5\pi}{24}\right)$

1.11　(3) $\cos\dfrac{\pi}{5}=\dfrac{1+\sqrt{5}}{4},\ \sin\dfrac{\pi}{5}=\dfrac{\sqrt{10-2\sqrt{5}}}{4}$　(4) $\cos\dfrac{\pi}{10}=\dfrac{\sqrt{10+2\sqrt{5}}}{4},\ \sin\dfrac{\pi}{10}=\dfrac{\sqrt{5}-1}{4}$

1.12　(1) $-32768i$　(2) -1　(3) $\dfrac{-1+i}{16}$　(4) $648\big(-1+\sqrt{3}i\big)$　(5) $\dfrac{\sqrt{3}+i}{64}$

(6) $-\dfrac{(97-56\sqrt{3})(1+\sqrt{3}i)}{2}$

1.13　(1) $\dfrac{\pi}{4}$

1.14 (1) $\dfrac{\pi}{4}$

1.16 (1) $\pm\dfrac{\sqrt{6}\,(1-i)}{2}$ (2) $\pm\sqrt[4]{3}\,(1+\sqrt{3}\,i)$ (3) $\pm\sqrt{5}\,(\sqrt{3}+i)$

(4) $\pm\dfrac{1}{2}\big\{\big(\sqrt{6}-\sqrt{2}\,\big)+i\big(\sqrt{6}+\sqrt{2}\,\big)\big\}$ (5) $\dfrac{\pm\sqrt{3}+i}{2}$，$-i$ (6) $\dfrac{\sqrt[3]{4}\,(\pm\sqrt{3}-i)}{2}$，$\sqrt[3]{4}i$

(7) $\dfrac{\sqrt[3]{4}\,(1+i)}{2}$，$\dfrac{\sqrt[3]{4}}{4}\left\{-(\sqrt{3}+1)+i(\sqrt{3}-1)\right\}$，$\dfrac{\sqrt[3]{4}}{4}\left\{(\sqrt{3}-1)-i(\sqrt{3}+1)\right\}$

(8) $\dfrac{\sqrt[6]{18}}{4}\big\{\big(\sqrt{6}+\sqrt{2}\,\big)+i\big(\sqrt{6}-\sqrt{2}\,\big)\big\}$，$\dfrac{\sqrt[6]{18}\,(-1+i)}{\sqrt{2}}$，$-\dfrac{\sqrt[6]{18}}{4}\big\{\big(\sqrt{6}-\sqrt{2}\,\big)+i\big(\sqrt{6}+\sqrt{2}\,\big)\big\}$

(9) $\pm(1+i)$，$\pm(1-i)$ (10) $\pm\dfrac{\sqrt[4]{2}\,(\sqrt{3}-i)}{2}$，$\pm\dfrac{\sqrt[4]{2}\,(1+\sqrt{3}i)}{2}$

(11) $\pm\dfrac{1}{2}\big\{\big(\sqrt{6}+\sqrt{2}\,\big)-i\big(\sqrt{6}-\sqrt{2}\,\big)\big\}$，$\pm\dfrac{1}{2}\big\{\big(\sqrt{6}-\sqrt{2}\,\big)+i\big(\sqrt{6}+\sqrt{2}\,\big)\big\}$

(12) 1，$\dfrac{1}{4}\left\{(\sqrt{5}-1)\pm i\sqrt{10+2\sqrt{5}}\right\}$，$\dfrac{1}{4}\left\{-(\sqrt{5}+1)\pm i\sqrt{10-2\sqrt{5}}\right\}$

(13) $\sqrt{3}\pm i$，$\pm 2i$，$-\sqrt{3}\pm i$

1.19 (1)，(6)-(9) は絶対収束.

(2) $k=1$ のとき発散，$k\ge 2$ のとき絶対収束.

(3) $|\beta|>1$ のとき発散，$|\beta|\le 1$ のとき絶対収束.

(4) $|\beta|\ge 1$ のとき発散，$|\beta|<1$ のとき絶対収束.

(5) $|\beta|\ge 1$ のとき発散，$|\beta|<1$ のとき絶対収束.

(10) $|\beta|\ge 1$ のとき発散，$|\beta|<1$ のとき絶対収束.

1.22 (1) $e^2\cos 3+ie^2\sin 2$ (2) $\dfrac{1}{\sqrt{e}}\big\{\cos(\ln 3)+i\sin(\ln 3)\big\}$ (3) $\sqrt{2}(1+i)$ (4) $\dfrac{\cos 2-i\sin 2}{e^2}$

(5) $\cos\dfrac{1}{2}-i\sin\dfrac{1}{2}$

1.23 (1) $\ln 5+2n\pi i$ (2) $\dfrac{1}{2}\ln 2+(2n+1)\pi i$ (3) $\ln 2+\left(n\pi-\dfrac{\pi}{4}\right)i$ (4) $\dfrac{3}{2}\ln 2+\left(2n\pi-\dfrac{\pi}{4}\right)i$

(5) $-\ln 2\sqrt{3}+\left(2n\pi-\dfrac{5\pi}{6}\right)i$ (6) $\dfrac{2i}{(4n-1)\pi}$，あるいは $-\dfrac{2i}{(4n+1)\pi}$

(7) $\pm\sqrt{n\pi+\dfrac{\pi}{3}}\cdot(1+i)$ $(n\ge 0)$，$\pm\sqrt{n\pi-\dfrac{\pi}{3}}\cdot(1-i)$ $(n\ge 1)$

1.25 (1)

$$
\begin{aligned}
n=4 \text{ のとき，}\quad & \cos^4\theta == \frac{1}{8}\big\{\cos 4\theta+4\cos 2\theta+3\big\},\\
n=5 \text{ のとき，}\quad & \cos^5\theta = \frac{1}{16}\big\{\cos 5\theta+5\cos 3\theta+10\cos\theta\big\},\\
n=6 \text{ のとき，}\quad & \cos^6\theta = \frac{1}{32}\big\{\cos 6\theta+6\cos 4\theta+15\cos 2\theta+10\big\}.
\end{aligned}
$$

(2)　$n=2m$ のとき，$\sin^{2m}\theta = \dfrac{(-1)^m}{2^{2m}}\displaystyle\sum_{k=0}^{2m}\binom{2m}{k}(-1)^k\cos\{(n-2k)\theta\}$,

$n=2m+1$ のとき，$\sin^{2m+1}\theta = \dfrac{(-1)^m}{2^{2m+1}}\displaystyle\sum_{k=0}^{2m+1}\binom{2m+1}{k}(-1)^k\sin\{(n-2k)\theta\}$.

1.27　(1) $\boldsymbol{X}=\{8,0,0,0,-8,0,0,0\}$　(2) $\boldsymbol{X}=\{4,0,0,0,4,0,0,0\}$

第 2 章　**2.1**　(1), (2) 微分可能な点はない.

(3) 原点のみで微分可能．$f'(0)=0$．あらゆる点で正則ではない.

(4) $\mathbb{C}$ 上で正則，$f'(x+iy)=2(x+iy)$ $(=2z)$

(5) 原点のみで微分可能．$f'(0)=0$．あらゆる点で正則ではない.

(6) 実軸上でのみ微分可能，$f'(x+i0)=2x$．あらゆる点で正則ではない.

(7) 放物線 $y=\dfrac{3}{2}x^2$ 上で微分可能，$f'\big(x+i\frac{3x^3}{2}\big)=6x^2$．あらゆる点で正則ではない.

(8) 2 直線 $y=\pm x-1$ 上で微分可能，$f'\big(x+i(\pm x-1)\big)=3x^2$．あらゆる点で正則ではない.

(9) 直線 $x=1$ および虚軸上で微分可能，$f'(x+iy)=0$．あらゆる点で正則ではない.

(10) あらゆる点で微分可能ではない.

(11) $\mathbb{C}$ 上で正則，$f'(x+iy)=2i(x+iy)e^{-2xy}\big\{\cos(x^2-y^2)+i\sin(x^2-y^2)\big\}$ $(=2iz\exp(iz^2))$

(12) $a=b=-1$ のとき，$f(z)$ は整関数で，$f'(z)=-\sin(x+iy)+i\cos(x+iy)=ie^{iz}$.

n を整数とする．$a=-1,\, b\neq -1$ のとき，$z=n\pi$ のみで微分可能，$f'(n\pi)=(-1)^n i$.

$a\neq -1,\, b=-1$ のとき，$z=n\pi+\dfrac{\pi}{2}$ で微分可能，$f'\left(n\pi+\dfrac{\pi}{2}\right)=(-1)^{n+1}$.

$a\neq -1,\, b\neq -1$ のとき，$|a+1|<|b+1|$ ならば $z=n\pi+\dfrac{i}{2}\ln\dfrac{b-a}{a+b+2}$ で微分可能,

$$f'(z)=i\frac{(-1)^n|b+1|}{\sqrt{(b-a)(a+b+2)}}\left\{\frac{b}{2}\ln\frac{b-a}{a+b+2}+1\right\}$$

$|a+1|>|b+1|$ ならば $z=\left(n\pi+\dfrac{\pi}{2}\right)+\dfrac{i}{2}\ln\dfrac{a-b}{a+b+2}$ で微分可能,

$$f'(z)=\frac{(-1)^{n+1}|a+1|}{\sqrt{(a-b)(a+b+2)}}\left\{\frac{a}{2}\ln\frac{a-b}{a+b+2}+1\right\}$$

$|a+1|=|b+1|$ ならばあらゆる点で微分可能ではない.

2.3　(1) (i) 偏微分可能でないので全微分可能ではない．(ii) 全微分可能.

(2) (i) 偏微分可能でない全微分可能ではない．(ii) 全微分可能.

(3) (i) $D_1 = \{0 < y < x\} \cup \{x < y < 0\}$ 上では正則，$f'(z) = 2z$.

(ii) $D_2 = \{0 < x < y\} \cup \{y < x < 0\}$ 上では微分可能ではない.

(iii) $D_3 = \{0 < -y < x\} \cup \{x < -y < 0\}$ 上では微分可能ではない.

(iv) $D_4 = \{0 < -x < y\} \cup \{-x < y < 0\}$ 上では正則，$f'(z) = -2z$.

(v) $z = 0$ で微分可能，$f'(0) = 0$．$z = 0$ では正則ではない.

2.4 (1)

$$y > 0 \text{ のとき，} \qquad f'(x+iy) = \frac{1}{\sqrt{x+\sqrt{x^2+y^2}} + i\sqrt{-x+\sqrt{x^2+y^2}}} = \frac{1}{f(z)},$$
$$y < 0 \text{ のとき，} \qquad f'(x+iy) = \frac{1}{\sqrt{x+\sqrt{x^2+y^2}} - i\sqrt{-x+\sqrt{x^2+y^2}}} = \frac{1}{f(z)}.$$

(2) $f'(z) = \dfrac{1}{z}$.

2.16 両者は一致する.

2.17 $\left(z^{1/n}\right)^m = z^{m/n}$ は成り立つ．$(m,n) = (2,4)$ のとき，$(z^m)^{1/n} = (z^2)^{1/4} \supsetneq z^{m/n} = z^{1/2}$ となるように，一般的には $(z^m)^{1/n} \neq z^{m/n}$．$(m,n) = (3,4)$ のとき，3 つは一致する.

2.18 (1) $\ln\bigl(\sqrt{17}-4\bigr) + \pi i$ (2) $-\dfrac{1}{2}\ln 2 + i\left(2n\pi - \dfrac{\pi}{4}\right)$ (3) $\dfrac{1}{2}\ln 24 + i\left(2n\pi - \dfrac{11\pi}{12}\right)$

(4) $4\ln 2 + i\left(2n\pi - \dfrac{2\pi}{3}\right)$ (5) $3 + i(2n\pi + 2)$ (6) $3 + (2\pi - 4)i$

(7) $e^{-\pi/4+2n\pi}\left\{\cos\left(\dfrac{1}{2}\ln 2\right) + i\sin\left(\dfrac{1}{2}\ln 2\right)\right\}$ (8) $e^{\pi/4+2n\pi}\left\{\cos\left(\dfrac{1}{2}\ln 2\right) - i\sin\left(\dfrac{1}{2}\ln 2\right)\right\}$

(9) $\dfrac{e^{6n\pi}}{25}\left\{\cos\bigl(3\ln 5\bigr) + i\sin\bigl(3\ln 5\bigr)\right\}$ (10) $3^{\sqrt{2}}\left[\cos\left\{(2n+1)\sqrt{2}\pi\right\} + i\sin\left\{(2n+1)\sqrt{2}\pi\right\}\right]$

(11) $\dfrac{1}{2}e^{2\pi/3+4n\pi}\left\{\cos\left(2\ln 2 + \dfrac{\pi}{3}\right) + i\sin\left(2\ln 2 + \dfrac{\pi}{3}\right)\right\}$

(12) $4e^{\pi^2(1/6+2n)}\left\{\cos\left(\ln 2 - \dfrac{1}{3}\right)\pi + i\sin\left(\ln 2 - \dfrac{1}{3}\right)\pi\right\}$

(13) $\exp\left[-\pi^2\left(\dfrac{1}{8} + m + \dfrac{n}{2} + 4mn\right)\right]\left[\cos\left\{\left(\dfrac{\pi}{4} + n\pi\right)\ln 2\right\} + i\sin\left\{\left(\dfrac{\pi}{4} + 4n\pi\right)\ln 2\right\}\right]$

2.19 (1) $\arcsin x$ (2) $\pi - \arcsin x$ $(0 \le x \le 1)$，$-(\pi + \arcsin x)$ $(-1 \le x < 0)$

(3) $\arccos x$ (4) $-\arccos x$

2.20 (1) $-\pi < \operatorname{Arg}\left(\dfrac{1+ix}{1-ix}\right) < \pi$ (2) $f(x) = \arctan x$，$f'(x) = \dfrac{1}{x^2+1}$

2.21 $\cosh(x+iy) = \cosh x\cos y + i\sinh x\sin y$，$\sinh(x+iy) = \sinh x\cos y + i\cosh x\sin y$，

$$\tanh(x+iy) = \frac{\sinh 2x + i\sin 2x}{2(\sinh^2 x + \cos^2 y)}$$

2.28 $\sinh z$：$z = n\pi i$，$\cosh z$：$z = \left(n\pi + \dfrac{\pi}{2}\right) i$，ただし n は整数.

2.31 $z \neq n\pi + \dfrac{\pi}{2}$ のとき，$u = \dfrac{\sin 2x}{2(\cos^2 x + \sinh^2 y)}$，$v = \dfrac{\sinh 2y}{2(\cos^2 x + \sinh^2 y)}$

2.33 $\dfrac{\sin^2 2x + \cosh^2 2y}{4(\cos^2 x + \sinh^2 y)^2}$

2.34 たとえば $n = \dfrac{1}{2}$ のとき成立しない.

2.35 (1) $z = 2n\pi$ (2) $z = 2n\pi - \dfrac{\pi}{2}$ (3) $z = 2n\pi \pm i \ln\left(4 + \sqrt{15}\right)$

(4) $z = \left(2n\pi - \dfrac{\pi}{2}\right) \pm i \ln\left(\sqrt{7} + \sqrt{6}\right)$ (5) $z = \left(n\pi + \dfrac{\pi}{4}\right) \pm i \dfrac{1}{2} \ln\left(5 + 2\sqrt{6}\right)$

(6) $z = \left(n\pi + \dfrac{\pi}{2}\right) + i(-1)^{n+1} \ln\left(\sqrt{5} + 2\right)$ (7) $z = \left(\dfrac{n\pi}{3} + \dfrac{\pi}{6}\right) + i \dfrac{(-1)^n}{3} \ln(\sqrt{17} + 4)$

(8) $z = n\pi + i(-1)^{n+1} \ln\left(\sqrt{37} + 6\right)$

2.36 (1) $z = (-1)^n \arcsin u + n\pi$ (2) $z = \pm \arccos u + 2n\pi$

2.37 $z = \left(n\pi + \dfrac{\pi}{2}\right) + i(-1)^{n+1} \ln\left(v + \sqrt{v^2 + 1}\right)$

2.38 $z = \left(2n\pi - \dfrac{\pi}{2}\right) \pm i \ln\left(-u + \sqrt{u^2 - 1}\right)$

2.39 $z = \arctan u + n\pi$

2.43 (1) $3xy^2 - x^3$ (2) $(x + y)(4xy - x^2 - y^2)$ (3) $e^x(x \sin y + y \cos y)$ (4) $-\dfrac{\sinh y}{\cosh y - \cos x}$

第 3 章 **3.1** (1) $I = \dfrac{\pi i}{2}$ (2) $I = \dfrac{\pi i}{2}$ (3) $I = i$ (4) $C : z = (1 - t) + it,\ t : 0 \to 1$，$I = i$

(5) $C : z = (1 - \sqrt{t}) + it,\ t : 0 \to 1$，$I = \dfrac{2i}{3}$

3.2 (1) $C : z = t,\ t : -1 \to 1$，$I = 1$ (2) $C : z = e^{it},\ t : \pi \to 0$，$I = 2$

(3) $C : z = e^{it},\ t : -\pi \to 0$，$I = 2$

(4) $C_1 : z = t + (1 + t)i,\ t :\ -1 \to 0$，$C_2 : z = t + (1 - t)i,\ t : 0 \to 1$，$I = 1 + \dfrac{\sqrt{2}}{2} \ln\left(1 + \sqrt{2}\right)$

3.3 (1) $I = \dfrac{\sqrt{3}}{2} + i\left(\dfrac{1}{2} + \dfrac{\sqrt{3}\pi}{12}\right)$ (2) $C : z = t + 2ti,\ t : 0 \to 1$，$I = 1 + 2i$

(3) $C : z = (1 - \pi i)t,\ t : 0 \to 1$，$I = \dfrac{e + 1}{\pi^2 + 1}\left\{(\pi^2 - 1) + 2\pi i\right\}$

(4) $C : z = (1 + t) + it,\ t : 0 \to 1$，$I = \dfrac{8}{3} + 6i$

(5) $C : z = \cos t + i(1 + \sin t),\ t :\ -\dfrac{\pi}{2} \to 0$，$I = 2 - \dfrac{\pi}{2} + i$

(6) $C: z=\cos t+i(1+\sin t),\ t:0\to\dfrac{\pi}{2}$， $I=-\dfrac{5}{6}+\dfrac{4+3\pi}{12}\,i$

(7) $C_1: z=t,\ t:1\to R$， $C_2: z=Re^{it},\ t:0\to\Theta$， $I=\left\{\ln R+\dfrac{1}{2}\bigl(\cos 2\Theta-1\bigr)\right\}+\dfrac{i}{2}\sin 2\Theta$

(8) $C\colon z=re^{it}+\alpha,\ t\colon 2n\pi\to 0$， $m=1$ のとき $I=-2n\pi i$， $m\neq 1$ のとき $I=0$.

3.4 (1) $I=\dfrac{\pi}{\sqrt{2}}$ (2) $I=-\dfrac{\pi}{\sqrt{2}}$ (3) $I=0$

3.5 (1) (i) $\dfrac{i}{2\sqrt{3}}\ln\bigl(2+\sqrt{3}\bigr)$ (ii) $\dfrac{1}{2\sqrt{3}}\left\{-2\arctan\dfrac{2}{\sqrt{3}}+i\ln\bigl(2+\sqrt{3}\bigr)\right\}$

(2) (i) $\dfrac{\pi}{2}+\dfrac{i}{2}\ln 3$ (ii) $-\dfrac{\pi}{2}+\dfrac{i}{2}\ln 3$ (iii) $\dfrac{\pi}{2}+\dfrac{i}{2}\ln 3$

3.12 (1) 0 (2) $2\pi i$ (3) $\dfrac{4\pi}{3}\sinh 2$ (4) $-\dfrac{27\pi(1+i)}{4096\sqrt{2}}$ (5) $\dfrac{3\pi}{2}$

(6) $-\dfrac{\pi}{3}\left(\sinh\dfrac{\sqrt{3}}{2}\sin\dfrac{1}{2}-\sqrt{3}\cosh\dfrac{\sqrt{3}}{2}\cos\dfrac{1}{2}\right)+\dfrac{\pi i}{3}\left(-2\cos 1+\cosh\dfrac{\sqrt{3}}{2}\cos\dfrac{1}{2}+\sqrt{3}\sinh\dfrac{\sqrt{3}}{2}\sin\dfrac{1}{2}\right)$

(7) $2\pi i$ (8) $\dfrac{\pi\bigl(3\sqrt{3}-\pi\bigr)i}{6}$ (9) $-\pi i$

第 4 章 **4.1** (1) $\dfrac{1}{|\beta|}$ (2) $\dfrac{1}{\sqrt{|\beta|}}$ (3) $\dfrac{1}{\sqrt[3]{|\beta|}}$ (4) 1 (5) e (6) $\dfrac{1}{3}$ (7) 1 (8) 2 (9) e^{-2}

(10) 0 ($|\beta|>1$)， $\dfrac{1}{|\gamma|}$ ($|\beta|=1$)， $+\infty$ ($|\beta|<1$) (11) 0 ($p>3$)， 27 ($p=3$)， $+\infty$ ($p<3$)

(12) $\dfrac{1}{2}$ (13) e^2

4.5 $\displaystyle\sum_{n=0}^{\infty}\frac{1}{(2n)!}\,z^{2n}$

4.6 $\displaystyle\sum_{n=0}^{\infty}\frac{1}{(2n+1)!}\,z^{2n+1}$

4.7 (1) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{(4n+3)\cdot(2n+1)!}\,z^{4n+3}$ (2) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{(4n+1)\cdot(2n)!}\,z^{4n+1}$

4.8 $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2\cdot(2n)!}\,z^{2n+1}$

4.9 (1) $\displaystyle\sum_{n=0}^{\infty}\frac{ei}{n!}\left\{z-\left(1+\frac{\pi i}{2}\right)\right\}^n$， $R=+\infty$

(2) $\displaystyle\frac{\sqrt{2}}{2}\left\{\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\left(z-\frac{\pi}{4}\right)^{2n}-\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\left(z-\frac{\pi}{4}\right)^{2n+1}\right\}$， $R=+\infty$

(3) $\dfrac{\pi i}{2} - \displaystyle\sum_{n=1}^{\infty} \frac{i^n}{n}(z-i)^n$, $R=1$ (4) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2 \cdot 4^{n-1}}{(2n)!} z^{2n}$, $R=+\infty$

(5) $\mathrm{Log}(2+i) + \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(2+i)^n}(z-i)^n$, $R=\sqrt{5}$ (6) $i\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n(2n+1)} z^{2n+1}$, $R=2$

(7) $\displaystyle\sum_{n=0}^{\infty}(-1)^n z^{3n} + \sum_{n=0}^{\infty}(-1)^n z^{3n+1}$, $R=1$

(8) $(1-i)\left\{ i + \dfrac{1}{4}(z-2i) + \displaystyle\sum_{n=2}^{\infty} \frac{4i^{n-1}(2n-3)!}{8^n n!(n-2)!}(z-2i)^n \right\}$, $R=2$

(9) $\cosh\dfrac{\pi}{4} \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\left(z-\frac{\pi i}{4}\right)^{2n+1} + i\sinh\frac{\pi}{4}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\left(z-\frac{\pi i}{4}\right)^{2n}$, $R=+\infty$

(10) $-\dfrac{1}{4}\displaystyle\sum_{n=0}^{\infty} \frac{(-i)^n(n+1)z^n}{2^n}$, $R=2$ (11) $\dfrac{1}{2}\displaystyle\sum_{n=0}^{\infty}(-2)^n(n+1)(n+2)z^n$, $R=\dfrac{1}{2}$

(12) $-\displaystyle\sum_{n=0}^{\infty}(-i)^n(n+1)(z+i)^n$, $R=1$ (13) $\dfrac{1}{2}\displaystyle\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{3^{n+1}}-1\right)(z+2i)^n$, $R=1$

(14) $\dfrac{1}{2\sqrt{2}}\displaystyle\sum_{n=0}^{\infty}(-1)^n\left\{ \frac{\left(2-\sqrt{2}-i\right)^{n+1}}{\left(7-4\sqrt{2}\right)^{n+1}} - \frac{\left(2+\sqrt{2}-i\right)^{n+1}}{\left(7+4\sqrt{2}\right)^{n+1}} \right\}(z-2)^n$, $R=7-4\sqrt{2}$

(15) $\dfrac{1}{3}\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)3^n} z^{2n+1}$, $R=\sqrt{3}$

(16) $-\dfrac{i}{4}\ln 3 - \dfrac{1}{4}\displaystyle\sum_{n=1}^{\infty}\frac{i^{n-1}}{n}\left\{1+\frac{(-1)^{n-1}}{3^n}\right\}(z+i)^n$, $R=1$

4.10 $f^{(n)}(x) = \left(\sqrt{2}\right)^n e^x \cos\left(x+\dfrac{n\pi}{4}\right)$, $g^{(n)}(x) = \left(\sqrt{2}\right)^n e^x \sin\left(x+\dfrac{n\pi}{4}\right)$,

$\alpha = 1+\sqrt{3}i$ と選んで，$h^{(n)}(x) = 2^n e^x \sin\left(x+\dfrac{n\pi}{3}\right)$.

4.11 $\displaystyle\sum_{n=0}^{\infty}\left\{\sum_{m=0}^{[n/2]}\frac{(-1)^m}{(n-2m)!(2m)!}\right\} x^n$，$[y]$ はガウス記号.

4.12 $\displaystyle\sum_{n=0}^{\infty}\left\{\sum_{m=0}^{[(n-1)/2]}\frac{(-1)^m}{(n-2m-1)!(2m+1)!}\right\} x^n$，$[y]$ はガウス記号.

4.13 $f^{(2n)}(0)=0$，$f^{(2n+1)}(0)=(-1)^n(2n)!$

4.14 $a_n = \dfrac{1}{\sqrt{5}}\left\{\left(\dfrac{\sqrt{5}+1}{2}\right)^n - \left(\dfrac{1-\sqrt{5}}{2}\right)^n\right\}$

4.17 (1) $\displaystyle\sum_{n=1}^{\infty}\tau(n)z^n$，$\tau(n)$ は n の約数の個数を表す.

(2) $\displaystyle\sum_{n=1}^{\infty}\sigma(n)z^n$，$\sigma(n)$ は n の約数の総和を表す.

第 5 章　5.1　(1) $f(z) = \frac{1}{z^9} - \frac{1}{6}\frac{1}{z^5} + \frac{1}{120}\frac{1}{z} + \sum_{n=3}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{4n-5}$, $R = +\infty$,

$P(z) = \frac{1}{z^9} - \frac{1}{6}\frac{1}{z^5} + \frac{1}{120}\frac{1}{z}$

(2) $f(z) = \frac{1}{2}\frac{1}{z^5} - \frac{1}{24}\frac{1}{z^3} + \frac{1}{720}\frac{1}{z} - \sum_{n=4}^{\infty} \frac{(-1)^{n+1}}{(2n)!} z^{2n-7}$, $R = \infty$,

$P(z) = \frac{1}{2}\frac{1}{z^5} - \frac{1}{24}\frac{1}{z^3} + \frac{1}{720}\frac{1}{z}$

(3) $f(z) = \frac{\ln 2 + \frac{\pi i}{2}}{z^5} - \frac{i}{2}\frac{1}{z^4} + \frac{1}{8}\frac{1}{z^3} + \frac{i}{24}\frac{1}{z^2} - \frac{1}{64}\frac{1}{z} - \sum_{n=5}^{\infty} \frac{i^n}{n2^n} z^{n-5}$, $R = 2$,

$P(z) = \frac{\ln 2 + \frac{\pi i}{2}}{z^5} - \frac{i}{2}\frac{1}{z^4} + \frac{1}{8}\frac{1}{z^3} + \frac{i}{24}\frac{1}{z^2} - \frac{1}{64}\frac{1}{z}$

(4) $f(z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)\cdot(2n+1)!} z^{2n+1}$, $R = +\infty$, $P(z) = 0$

(5) $f(z) = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!}\frac{1}{z^{2n+1}}$, $R = +\infty$, $P(z) = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!}\frac{1}{z^{2n+1}}$

(6) $f(z) = \frac{1}{2z^4} + \frac{1}{4z^3} + \frac{1}{8z^2} + \frac{1}{16z} + \sum_{n=4}^{\infty} \frac{z^{n-4}}{2^{n+1}}$, $R = 2$, $P(z) = \frac{1}{2z^4} + \frac{1}{4z^3} + \frac{1}{8z^2} + \frac{1}{16z}$

(7) $f(z) = \frac{1}{z^7} - \frac{2}{z^6} + \frac{3}{z^5} - \frac{4}{z^4} + \frac{5}{z^3} - \frac{6}{z^2} + \frac{7}{z} + \sum_{m=7}^{\infty} (-1)^m (m+1) z^{m-7}$, $R = 1$,

$P(z) = \frac{1}{z^7} - \frac{2}{z^6} + \frac{3}{z^5} - \frac{4}{z^4} + \frac{5}{z^3} - \frac{6}{z^2} + \frac{7}{z}$

5.2　(1) (i) $\sum_{n=0}^{\infty} \frac{(-i)^n}{2}\left(\frac{1}{3^{n+1}} - 1\right) z^n$　(ii) $\frac{1}{2}\sum_{n=0}^{\infty} \frac{(-i)^n}{3^{n+1}} z^n + \frac{1}{2}\sum_{m=1}^{\infty} \frac{i^m}{z^m}$　(iii) $\sum_{m=2}^{\infty} \frac{i^m(1-3^{m-1})}{2}\frac{1}{z^m}$

(2) (i) $\sum_{n=0}^{\infty} \frac{2^{n+1}-1}{2^{n+1}} (z-1)^n$　(ii) $-\sum_{n=0}^{\infty} \frac{1}{2^{n+1}} (z-1)^n - \sum_{m=1}^{\infty} \frac{1}{(z-1)^m}$

(3) (i) $\sum_{n=0}^{\infty} \frac{(-i)^n}{2}\left(\frac{1}{3^{n+1}} - 1\right)(z+i)^n$　(ii) $\frac{1}{2}\sum_{n=0}^{\infty} \frac{(-i)^n}{3^{n+1}} (z+i)^n + \frac{1}{2}\sum_{m=1}^{\infty} \frac{i^m}{(z+i)^m}$

(iii) $\sum_{m=2}^{\infty} \frac{i^m(1-3^{m-1})}{2}\frac{1}{(z+i)^m}$

(4) (i) $\frac{1}{4}\left(\frac{1}{z^3} + \frac{1}{z^2} + \frac{3}{4}\frac{1}{z}\right) + \frac{1}{4}\sum_{n=3}^{\infty} \frac{n+1}{2^n} z^{n-3}$

(ii) $\frac{1}{16}\left\{\frac{1}{(z-2)^2} - \frac{3}{z-2}\right\} + \frac{1}{16}\sum_{n=2}^{\infty} \frac{(-1)^n(n+2)(n+1)}{2^n} (z-2)^{n-2}$

(5) (i) $-\frac{1}{4}\left\{\frac{1}{(z-i)^2} + \frac{i}{z-i}\right\} - \frac{1}{4}\sum_{n=2}^{\infty} \frac{(n+1)i^n}{2^n} (z-i)^{n-2}$

(ii) $-\frac{1}{4}\left\{\frac{1}{(z+i)^2} - \frac{i}{z+i}\right\} - \frac{1}{4}\sum_{n=2}^{\infty} \frac{(n+1)(-i)^n}{2^n} (z+i)^{n-2}$

5.4　$\frac{1}{z} + \sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n} B_{2n}}{(2n)!} z^{2n-1}$

5.10 (1) 例えば，$\left\{\left(2m\pi i\right)^{-1}\right\}$ (2) 例えば，$\left\{\left(2m\pi+\dfrac{i}{m}\right)^{-1}\right\}$

第 6 章

6.1 (1) $z=z_0$ は n 位の極，$n=1$ ならば $\mathrm{Res}(f,\,z_0)=1$，$n\geq 2$ ならば $\mathrm{Res}(f,\,z_0)=0$.

(2) $z=-2,\,1\pm\sqrt{3}i$ は 1 位の極，$\mathrm{Res}(f,\,-2)=\dfrac{1}{12e^2}$，
$\mathrm{Res}\big(f,\,1\pm\sqrt{3}i\big)=-\dfrac{e}{24}\left\{\big(\cos\sqrt{3}-\sqrt{3}\sin\sqrt{3}\,\big)\pm i\big(\sqrt{3}\cos\sqrt{3}+\sin\sqrt{3}\,\big)\right\}$

(3) $z=\pm\sqrt{2}i$ は 2 位の極，$\mathrm{Res}\big(f,\,\pm\sqrt{2}i\big)=0$

(4) $z=0$ は 2 位の極，$z=-i$ は 1 位の極，$\mathrm{Res}(f,\,0)=1+\dfrac{\pi i}{2}$，$\mathrm{Res}(f,\,-i)=-i$

(5) $z=-\dfrac{2}{3}$ は 3 位の極，$\mathrm{Res}\left(f,\,-\dfrac{2}{3}\right)=\dfrac{1}{27}$

(6) $z=0,\,-\dfrac{\pi}{2}$ は 1 位の極，$\mathrm{Res}(f,\,0)=\dfrac{1}{\pi}$，$\mathrm{Res}\left(f,\,-\dfrac{\pi}{2}\right)=-\dfrac{2}{\pi^2}$

(7) $z=0$ は 3 位の極，$\mathrm{Res}(f,\,0)=\dfrac{1}{120}$

(8) $n=1,2$ のとき $z=0$ は除去可能な特異点．$n\geq 3$ のときに $z=0$ は $n-2$ 位の極．n が偶数ならば $\mathrm{Res}(f,\,0)=0$，n が奇数：$n=2\ell+1$ ならば $\mathrm{Res}(f,\,0)=\dfrac{(-1)^\ell}{(2\ell)!}$.

(9) $z=0$ は孤立真性特異点，$\mathrm{Res}(f,0)=\dfrac{5}{6}$

(10) $z=0$ は孤立真性特異点，$\mathrm{Res}(f,\,0)=\dfrac{2^{n+1}}{(n+1)!}$

(11) $z=1,2$ のとき $z=0$ は除去可能な特異点．$n\geq 3$ のとき $z=0$ は $n-2$ 位の極．$n\neq 4m+3$ $(m=0,1,2,\ldots)$ ならば, $\mathrm{Res}(f,\,0)=0$．$n=4m+3\,(m=0,1,2,\ldots)$ ならば, $\mathrm{Res}(f,\,0)=\dfrac{(-1)^m}{(2m+1)!}$

(12) $z=0$ は 1 位の極，$\mathrm{Res}(f,\,0)=1$

(13) $z=\left(2m+\frac{1}{2}\right)\pi\pm i\ln\big(2+\sqrt{3}\big)$ は 1 位の極，$\mathrm{Res}\left\{f,\,\left(2m+\frac{1}{2}\right)\pi\pm i\ln\big(2+\sqrt{3}\big)\right\}=\pm\dfrac{i}{\sqrt{3}}$

(14) $z=\ln 3+2n\pi i$ は 1 位の極，$\mathrm{Res}\big(f,\,\ln 3+2n\pi i\big)=-\dfrac{1}{3}$

(15) $z=\pi$ は真性特異点，$\mathrm{Res}(f,\,\pi)=0$

(16) $z=0$ は除去可能な特異点，$\mathrm{Res}(f,\,0)=0$

(17) $z=0$ は真性特異点．n が偶数ならば $\mathrm{Res}(f,0)=0$，
n が奇数 $n=2m+1$ のとき $\mathrm{Res}\big(f,0\big)=\dfrac{(-1)^m3^m}{(2m)!}$.

(18) $z=0$ は真性特異点．n が奇数のとき $\mathrm{Res}\big(f,0\big)=0$，
$n=2m$（$m\geq 1$）のとき $\mathrm{Res}\big(f,0\big)=\dfrac{(-1)^{m-1}2^{m-1}}{(2m-1)!}$.

(19) $z=\dfrac{1}{2}\ln 2+\left(\dfrac{\pi}{4}+2n\pi\right)i$ は 1 位の極，$\mathrm{Res}\left\{f,\,\dfrac{1}{2}\ln 2+i\left(\dfrac{\pi}{4}+2n\pi\right)\right\}=\dfrac{1-i}{2}$

(20) $z=0$ は除去可能な特異点

6.2 (1) -4π (2) $\sqrt{2}\pi i \sin\dfrac{\sqrt{2}}{2}$ (3) $2\pi i$ (4) $\pi\sinh 2$ (5) $\dfrac{4\pi i}{9}$ (6) $\dfrac{3\pi i}{128}$ (7) $2\pi i$ (8) $\dfrac{2\pi i}{3}$
(9) $2\pi i$ (10) $-\dfrac{4\pi i}{25}$ (11) $-4\pi i$ (12) $-\dfrac{4ni}{\pi}$ (13) $2\pi i\bigl(1+e^{-\pi}\bigr)$ (14) $\dfrac{\pi i}{12}$ (15) 0 (16) 0
(17) $\dfrac{2\pi i}{(n+1)!}$ (18) $-\dfrac{(6+5i)\pi}{3}$

6.4 $\dfrac{\pi\cdot(2n)!}{2^{2n-1}\bigl(n!\bigr)^2}$

6.5 (1) $\dfrac{2\pi}{\sqrt{a^2-1}}$ (2) $\dfrac{\pi}{1-a^2}$ (3) $\dfrac{2\pi}{a(a^2-1)}$ (4) $\dfrac{2\pi}{\sqrt{5}}$ (5) π (6) $\dfrac{\pi}{2\sqrt{a^2+a}}$
(7) $\dfrac{\pi a}{(a^2-1)\sqrt{a^2-1}}$ (8) $\dfrac{a^n\pi}{1-a^2}$

(9) n が偶数のときにはゼロ，n が奇数：$n=2m+1$ のときには $\dfrac{(-1)^m 2\pi a^n}{1-a^2}$ (10) $\dfrac{\bigl(\sqrt{5}-\sqrt{3}\bigr)\pi}{4}$

(11) $\dfrac{(3\sqrt{2}-4)\pi}{16}$ (12) $\dfrac{\sqrt{2}\pi}{4a}$ (13) $\dfrac{5\pi}{2}$ (14) $\dfrac{\pi e^{-a}}{2}$ (15) $\dfrac{2e^{-\sqrt{3}}\pi\sin 1}{\sqrt{3}}$

(16) $\dfrac{\pi e^{-a\xi/\sqrt{2}}}{\sqrt{2}a^3}\left(\cos\dfrac{a\xi}{\sqrt{2}}+\sin\dfrac{a\xi}{\sqrt{2}}\right)$ (17) $\dfrac{\pi e^{-a\xi/\sqrt{2}}}{a^2}\sin\dfrac{a\xi}{\sqrt{2}}$ (18) $\dfrac{1}{4a}\pi(1-a\xi)e^{-a\xi}$

(19) $-\dfrac{1}{4}\pi e^{-a|\xi|}(2-a|\xi|)$

6.6 (1) $\dfrac{2\pi}{3a^5}$

6.7 $\dfrac{\pi(2n-2)!}{2^{2n-1}\{(n-1)!\}^2}$

6.8 (1) $\dfrac{\sqrt[3]{4}\pi}{\sqrt{3}}$ (2) $\dfrac{\pi}{6\sqrt{3}}$ (3) $\dfrac{\sqrt[3]{2}\pi}{9\sqrt[6]{243}}$ (4) $\dfrac{\pi(\ln a-1)}{4a^3}$

6.12 (1) $\dfrac{2\pi\ln a}{3\sqrt{3}a}$ (2) $\dfrac{\ln 2}{2}$ (3) $\dfrac{1}{2b}\arctan\dfrac{b}{a}\cdot\ln(a^2+b^2)$ (4) $\dfrac{\pi}{2a}(\ln a)^2+\dfrac{\pi^3}{8a}$
(5) $a\neq b$ のとき $\dfrac{\pi}{2ab(b^2-a^2)}\bigl\{b(\ln a)^2-a(\ln b)^2\bigr\}+\dfrac{\pi^3}{8ab(a+b)}$,

$a=b$ のとき $\dfrac{\pi}{4a^3}\bigl\{(\ln a)^2-2\ln a\bigr\}+\dfrac{\pi^3}{16a^3}$

6.13 $\dfrac{\pi^2}{4}$

6.14 $0<r<1$ のとき $I(r)=0$, $r>1$ のとき $I(r)=2\pi\ln r$